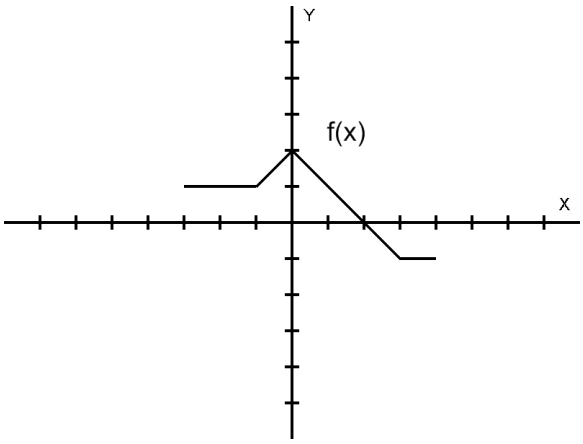
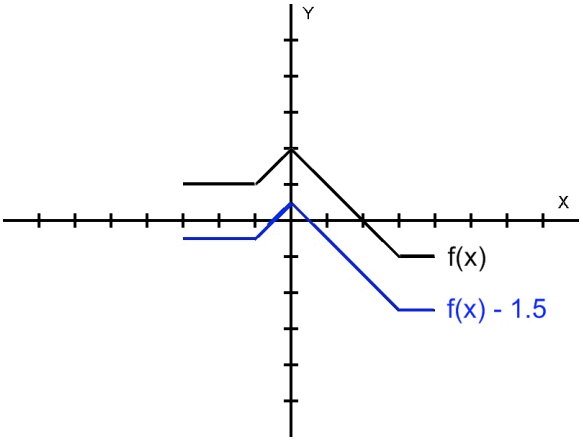
Transformation Basics

Transformations, in general, are when we take a function and manipulate it in such a way as to shift, stretch, or flip the graph of the function. There are three basic ways a graph can be changed; it can be shifted, it can be stretched/compressed, and it can be flipped. Let's start with a base function that we'll call f(x).

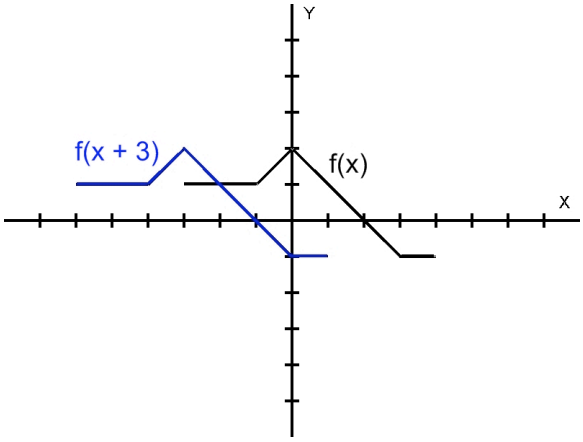


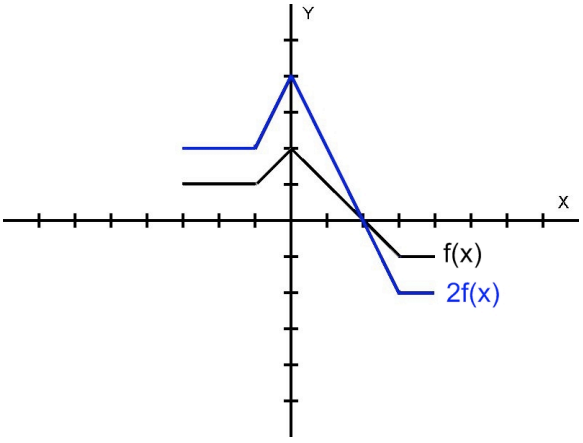
On the right we can see our base function, f(x). On the following graphs, the transformed function will be shown in blue.



On the left we can see that the graph has been shifted down one and a half units. This was done by subtracting one point five from f(x). Had we added one point five instead, the graph would have been shifted up. These are examples of vertical shifts.

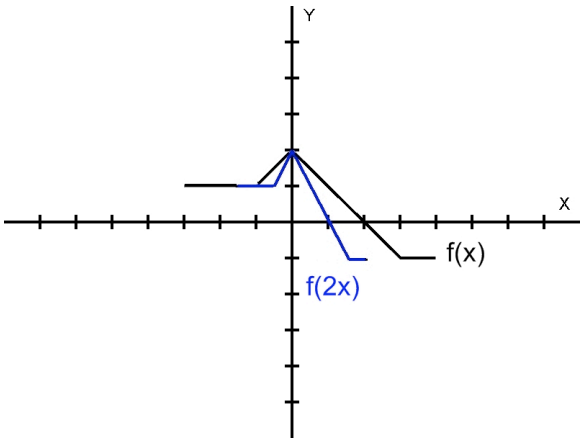
On the right we can see that the graph has been shifted to the left three units. This was done by adding three to x. Had we subtracted three instead, the graph would have been shifted to the right. Notice how these horizontal shifts are not quite as intuitive as the vertical shifts. Adding a number to x shifts the graph in the negative direction, while subtracting a number from x shifts the graph in the positive direction.

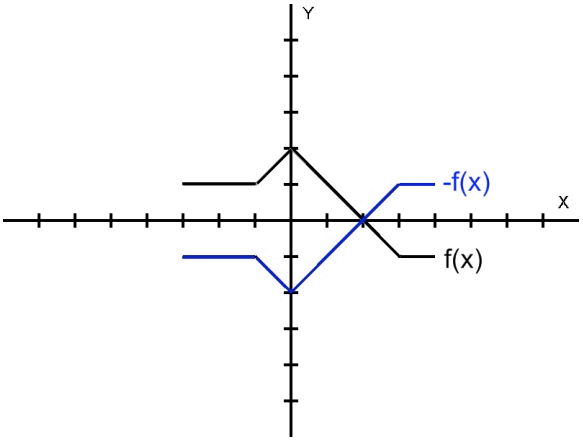




On the left we can see that the graph has been stretched vertically by a factor of two. This was done by multiplying f(x) by two. Had we divided instead, it would have compressed the graph by a factor of two. These are examples of vertical stretches/compressions.

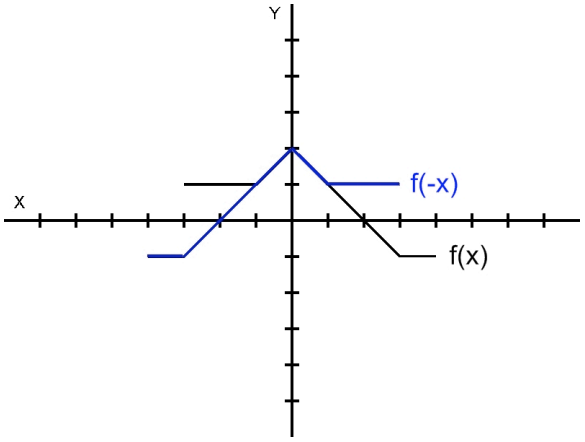
On the right we can see that the graph has been compressed horizontally by a factor of two. This was done by multiplying x by two. Had we divided instead, it would have stretched the graph instead. Notice how these horizontal stretches/compressions are not quite as intuitive as the vertical stretches/compressions. Multiplying x by a number compresses the graph horizontally, while dividing x by a number stretches the graph horizontally.

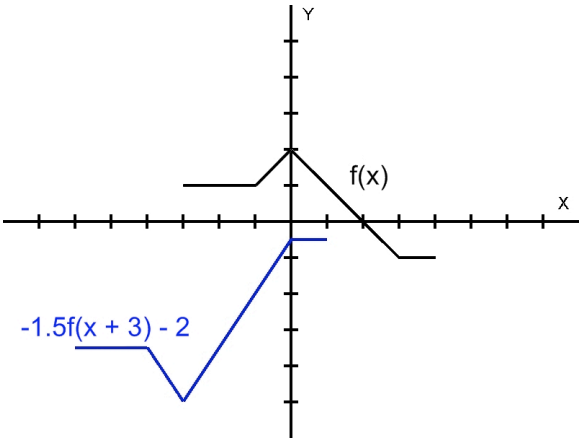




On the left we can see that the graph has been flipped over the x-axis. This was done by placing a negative sign in front of f(x).

On the right we can see that the graph has been flipped over the y-axis. This was done by replacing x with -x.





On the left we can see that the graph has had multiple transformations applied. Can you see what they are? The graph has been shifted left three units, flipped vertically, stretched vertically by a factor of one point five, and, finally, it has been shifted down two units. This was done by replacing f(x) with -1.5f(x + 3) – 2. **So**, for doing transformations, list the transformations, then apply them in the order of inside the function to outside the function. With -1.5f(x + 3) – 2 we can see x has been replaced with x + 3 (left three units), f(x +3) has been replaced with -1.5f(x + 3) (vertical flip and stretch by one point five), and -1.5f(x + 3) has been replaced with -1.5f(x + 3) – 2 (down two units). So perform the transformations in that order.