Logarithmic Functions

$y=\log_a x$ $x=a^y$ (exponential form)

Properties of Logarithms

1. $\log_a 1=0$ because $a^0=1$
2. $\log_a a=1$ because $a^1=a$
3. $\log_a a^x=x$ and $a^{\log_a x}=x$ Inverse Property
4. If $\log_a x=\log_a y$ then $x=y$ One-to-one

Natural Logarithms

$y=\ln x$ if $x=e^y$

Properties of Logarithms

1. $\ln 1=0$ because $e^0=1$
2. $\ln e=1$ because $e^1=e$
3. $\ln e^x=x$ and $e^{\ln x}=x$ inverse properties
4. If $\ln x=\ln y$ then $x=y$ one-to-one

Logarithmic Properties

1. Product—$\log_a(xy)=\log_a x+\log_a y$
2. Quotient—$\log_a(x/y)=\log_a x-\log_a y$
3. Power—$\log_a x^y=y\log_a x$

Natural Logarithmic Properties

1. Product—$\ln(xy)=\ln x+\ln y$
2. Quotient—$\ln(x/y)=\ln x-\ln y$
3. Power—$\ln x^y=y\ln x$

Change of Base

Base b	Base 10	Base e
$\log_a x=\frac{\log_b x}{\log_b a}$	$\log_a x=\frac{\log_{10} x}{\log_{10} a}$	$\text{Log}_a x=\frac{\ln x}{\ln a}$

Use the definition of Logarithmic Function to evaluate each logarithmic for indicated value of x

a. $f(x)=\log_2 x$, $x=32$

$y=\log_2 32$

$2^y=32$ exponential form

$2^y=2^5$

$y=5$

b. $f(x)=\log_{10} x$, $x=1/100$

$y=\log_{10}(1/100)$

$10^y=1/100$

$10^y=10^{-2}$

$y=-2$

Use calculator to evaluate the function

a. $\log_{10}10 = 1$
b. $\log_{10}2.5 = .3979400$
c. $\ln 2 = .6931472$
d. $\ln(-1)$ = ERROR domain of lnx is the set of positive real numbers, ln(-1) is undefined
e. $\log_{10}(-2)$ = ERROR domain of lnx is the set of positive real numbers, ln(-1) is undefined

(Note using a calculator can only be used with functions of base 10 or base *e*, also called the common logarithmic function, so you may need to use the Change of Base formula, as shown below.)

Changing base using common logarithms

a. $\log_4 25$

$$\frac{\log_{10}25}{\log_{10}4} \quad \text{Change of Base}$$

$$\frac{1.39794}{.60206} \approx 2.32$$

b. $\log_4 25$ (use Natural Logarithms)

$$\frac{\ln 25}{\ln 4}$$

$$\frac{3.21888}{1.386} \approx 2.32$$

Write each logarithm in terms of ln2 and ln3

a. ln6
ln(2 X 3)
ln2 + ln3 Product Property

b. ln(2/27)
ln2-ln27 Quotient Property
$\ln 2 - \ln 3^3$
ln2-3ln3 Power Rule

Expand or condense each expression

Expand

a. $\ln(\sqrt{3x-5} / 7)$
$\ln[(3x-5)^{1/2}/ 7]$
$\ln(3x-5)^{1/2}-\ln 7$ Quotient Property
$\frac{1}{2}\ln(3x-5)-\ln 7$ Power Property

Condense

b. $1/3[\log_2 x+\log_2(x-4)]$
$1/3[\log_2 x(x-4)]$ Product Property
$\log_2[x(x-4)]^{1/3}$ Power Property
$\log_2 \sqrt[3]{(x(x-4))}$